Παρατηρήσεις

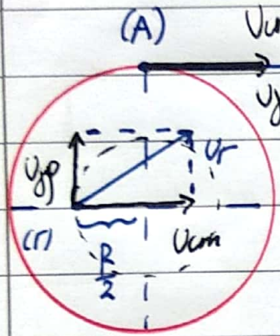
ΘΕΜΑ Α

Α1 γ **Α2** α **Α3** γ **Α4** δ

Α5 α) Σ β) Λ γ) Σ δ) Σ ε) Λ

ΘΕΜΑ Β

Β1 Σωστή απάντηση η (iii)



• Σημείο Α

$\upsilon_{cm} = \omega \cdot R$ (Κ.Χ.Ο.)

$\upsilon_{\gamma\rho} = \omega \cdot R = \upsilon_{cm}$

$\upsilon_A = \upsilon_{cm} + \upsilon_{\gamma\rho} \Rightarrow \upsilon_A = 2\cdot\upsilon_{cm}$ (1)

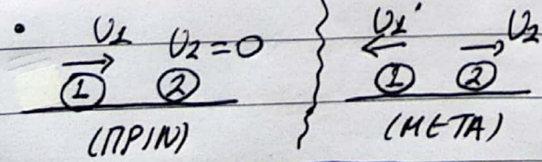
• Σημείο Γ

$\left.\begin{aligned} \upsilon_{cm} &= \omega\cdot R \text{ (Κ.Χ.Ο.)} \\ \upsilon_{\gamma\rho} &= \omega\cdot\frac{R}{2} = \frac{\upsilon_{cm}}{2} \end{aligned}\right\} \Rightarrow \upsilon_\Gamma = \sqrt{\upsilon_{cm}^2 + \upsilon_{\gamma\rho}^2} \Rightarrow \upsilon_\Gamma = \sqrt{\upsilon_{cm}^2 + \frac{\upsilon_{cm}^2}{4}}$

$\Rightarrow \upsilon_\Gamma = \sqrt{\frac{5\cdot\upsilon_{cm}^2}{4}} = \frac{\sqrt{5}\cdot\upsilon_{cm}}{2}$ (2)

$\frac{(2)}{(1)} \Rightarrow \frac{\upsilon_\Gamma}{\upsilon_A} = \frac{\frac{\sqrt{5}}{2}\cancel{\upsilon_{cm}}}{2\cdot\cancel{\upsilon_{cm}}} \Rightarrow \boxed{\frac{\upsilon_\Gamma}{\upsilon_A} = \frac{\sqrt{5}}{4}}$

Β2 Σωστή απάντηση η (ii)



$\Pi_1 = \frac{\Delta K_2}{K_1}\cdot 100\%$

$\Rightarrow \Pi_1 = \frac{K_2'}{K_1}\cdot 100\%$

$\Rightarrow \Pi_1 = \frac{\cancel{\frac{1}{2}} m_2 \cdot \upsilon_2'^2}{\cancel{\frac{1}{2}} m_1 \cdot \upsilon_1^2}\cdot 100\% = \frac{m_2\cdot\left(\frac{2m_1}{m_1+m_2}\right)^2\cdot\cancel{\upsilon_1^2}}{m_1\cdot\cancel{\upsilon_1^2}}\cdot 100\% \Rightarrow \Pi_1 = \frac{m_2}{m_1}\left(\frac{2m_1}{m_1+m_2}\right)^2\cdot 100\%$ (1)

Παρατηρήσεις

• $\overset{\vec{v}_2}{②}$ $\quad \overset{v_1=0}{①}$ $\Big\}$ $\overset{\vec{v}_2'}{②}$ $\quad \overset{\vec{v}_1'}{①}$

$$\pi_2 = \frac{\Delta K_1}{K_2}\cdot 100\% \Rightarrow \pi_2 = \frac{K_1'}{K_2}\cdot 100\%$$

$$\Rightarrow \pi_2 = \frac{\cancel{\frac{1}{2}} m_1 \cdot v_1'^2}{\cancel{\frac{1}{2}} m_2 \cdot v_2^2}\cdot 100\% = \frac{m_1 \left(\frac{2m_2}{m_1+m_2}\right)^2 \cdot \cancel{v_2^2}}{m_2 \cdot \cancel{v_2^2}}\cdot 100\%$$

$$\Rightarrow \boxed{\pi_2 = \frac{m_1}{m_2}\left(\frac{2m_2}{m_1+m_2}\right)^2 \cdot 100\%} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\pi_1}{\pi_2} = \frac{\frac{m_2}{m_1}\,\frac{(2m_1)^2}{\cancel{(m_1+m_2)^2}}\cdot \cancel{100\%}}{\frac{m_1}{m_2}\,\frac{(2m_2)^2}{\cancel{(m_1+m_2)^2}}\cdot \cancel{100\%}}$$

$$\Rightarrow \frac{\pi_1}{\pi_2} = \frac{m_2^2 \cdot 4 m_1^2}{m_1^2 \cdot 4 m_2^2} \Rightarrow \frac{\pi_1}{\pi_2} = 1$$

$$\Rightarrow \boxed{\pi_1 = \pi_2}$$

B3 Σωστή απάντηση η (i)

~> Οριζόντια βολή μέχρι το έδαφος:

$$S = v_0 \cdot t \Rightarrow S = v_0 \cdot \sqrt{\frac{2h_1}{g}} \quad (1)$$

~> Οριζόντια βολή μέχρι το Ζ:

$$\frac{S}{2} = v_0 \cdot t \Rightarrow \frac{S}{2} = v_0 \cdot \sqrt{\frac{2(h_1-h_2)}{g}} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{S}{\frac{S}{2}} = \frac{\cancel{v_0}\cdot\sqrt{\frac{2h_1}{g}}}{\cancel{v_0}\cdot\sqrt{\frac{2(h_1-h_2)}{g}}} \Rightarrow 2 = \sqrt{\frac{\cancel{2}h_1 \cdot \cancel{g}}{\cancel{2}(h_1-h_2)\cancel{g}}}$$

$$\Rightarrow \frac{h_1}{h_1-h_2} = 4 \Rightarrow h_1 = 4h_1 - 4h_2 \Rightarrow \boxed{h_2 = \frac{3h_1}{4}}$$

$$\Rightarrow h_1 = \frac{4}{3}\cdot\left(\frac{21H}{32}\right) \Rightarrow h_1 = \frac{7}{8}H \quad (3)$$

Εν Δυνάμει
ΦΡΟΝΤΙΣΤΗΡΙΑ

3

Παρατηρήσεις

→ Εξίσωση Bernoulli από την ελεύθερη επιφάνεια μέχρι το Ο:

$$\cancel{P_{atm}} + 0 + \cancel{\rho} \cdot g \cdot H = \cancel{P_{atm}} + \frac{1}{2} \cdot \cancel{\rho} \cdot v_0^2 + \cancel{\rho} \cdot g \cdot h$$

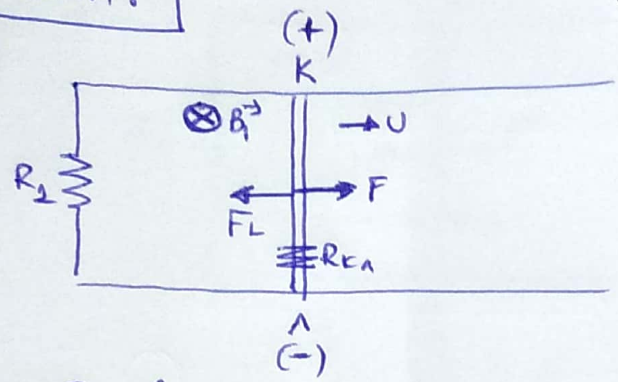
$$\Rightarrow g \cdot (H-h) = \frac{1}{2} v_0^2 \Rightarrow v_0 = \sqrt{2g(H-h)}$$

Άρα:

$$\Pi = A \cdot v_0 = A \cdot \sqrt{2g\left(H - \frac{7}{8}H\right)}$$

$$\Rightarrow \Pi = A \cdot \sqrt{2g\left(\frac{H}{8}\right)} \Rightarrow \boxed{\Pi = \frac{A}{2} \cdot \sqrt{gH}}$$

ΘΕΜΑ Γ



Γ1)

$E_{\epsilon\pi} = B_1 u \ell$, $I = \dfrac{B_1 u \ell}{R_1 + R_{KA}}$, $F_L = BI\ell$, $\Sigma F = ma \Rightarrow F - F_L = ma$

$u\uparrow \rightsquigarrow I\uparrow \rightsquigarrow F_L\uparrow \rightsquigarrow a\downarrow$ μέχρι $\Sigma F = 0$ οπότε $u = u_{op}$.

Η κίνηση είναι επιταχυνόμενη με μειούμενη επιτάχυνση μέχρι να αποκτήσει σταθερή οριακή ταχύτητα u_{op}.

Εύρεση u_{op}: $\Sigma F = 0 \Rightarrow F = F_L \Rightarrow F = B_1 I \ell \Rightarrow F = B_1 \dfrac{B_1 u_{op} \ell}{R_1 + R_{KA}} \cdot \ell$

$\Rightarrow F = \dfrac{B_1^2 u_{op} \cdot \ell^2}{R_1 + R_{KA}} \Rightarrow \boxed{u_{op} = 4 m/s}$

Γ2) Θα πρέπει $\Sigma F = 0 \Rightarrow F' = F_L \Rightarrow F' = B_3 I \cdot \ell \Rightarrow$

$F' = B_3 \dfrac{B_3 u_{op} \ell}{R_1 + R_{KA}} \cdot \ell \Rightarrow F' = \dfrac{B_3^2 u_{op} \cdot \ell^2}{R_1 + R_{KA}} \Rightarrow \boxed{F' = 0{,}8 N}$

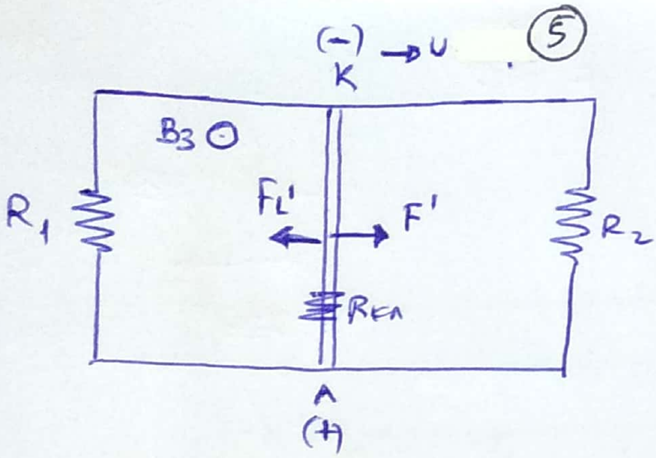
Γ3) $I = \dfrac{B_3 u_{op} \ell}{R_1 + R_{KA}} = 0{,}8 A$

Αφού $u = u_{op} = $ σταθ είναι και $I = 0{,}8 A = $ σταθ.

Άρα από t_2 σε t_3 : $I = \dfrac{Q_{\epsilon\pi\alpha\gamma}}{\Delta t} \Rightarrow \Delta t = \dfrac{Q_{\epsilon\pi\alpha\gamma}}{I} = \dfrac{0{,}2}{0{,}8} = \dfrac{1}{4} sec$

$Q_{R o\lambda} = I^2 R_{o\lambda} \Delta t = I^2 (R_1 + R_{KA}) \cdot \Delta t \Rightarrow \boxed{Q_{R o\lambda} = 0{,}8 J}$

Γ4)



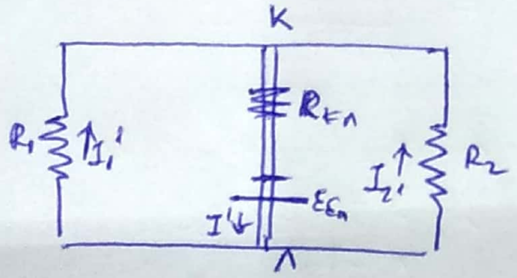
Για την νέα οριακή ταχύτητα $υ_{op}'$ έχουμε:

$\Sigma F = 0 \Rightarrow F' = F_L' \Rightarrow F' = B_3 I' \ell \Rightarrow F' = B_3 \dfrac{B_3 υ_{op}' \ell}{R_{ολ}'} \cdot \ell \Rightarrow$

$F' = \dfrac{B_3^2 υ_{op}' \cdot \ell^2}{R_{ολ}'} \Rightarrow υ_{op}' = 3{,}2\, m/s$

όπου $R_{ολ}' = R_{12} + R_{KΛ} = \dfrac{R_1 R_2}{R_1 + R_2} + R_{KΛ} = 1Ω + 3Ω = 4Ω$

$I' = \dfrac{B_3 υ_{op}' \cdot \ell}{R_{ολ}'} = 0{,}8 A$



$V_K + I_2' R_2 = V_Λ \Rightarrow V_K - V_Λ = -I_2' R_2 \Rightarrow$

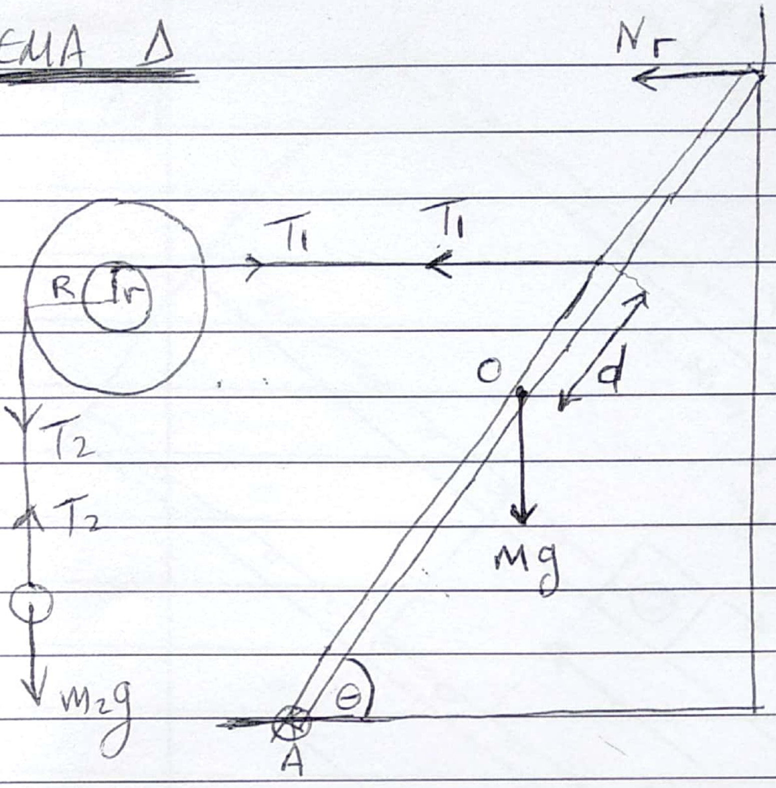
$V_{KΛ} = -I_2' R_2 = -0{,}8 V$

$V_1 = V_2 \Rightarrow I_1' R_1 = I_2' R_2 \Rightarrow I_1' = I_2'$

και $I' = I_1' + I_2'$

Άρα $I' = 2 I_2' \Rightarrow I_2' = \dfrac{I'}{2} = 0{,}4 A$

ΘΕΜΑ Δ



Δ1

$\Sigma F_y = 0 \Rightarrow T_2 = m_2 g = 30N$

$\Sigma \tau_{\tau\rho o\chi} = 0 \Rightarrow \tau_{T_1} = \tau_{T_2} \Rightarrow T_1 r = T_2 R$

$\Rightarrow T_1 = 2T_2 \Rightarrow T_1 = 60N$

δοκός $\quad \Sigma \tau_A = 0 \Rightarrow \tau_{N_\Gamma} + \tau_{T_1} - \tau_{Mg} = 0$

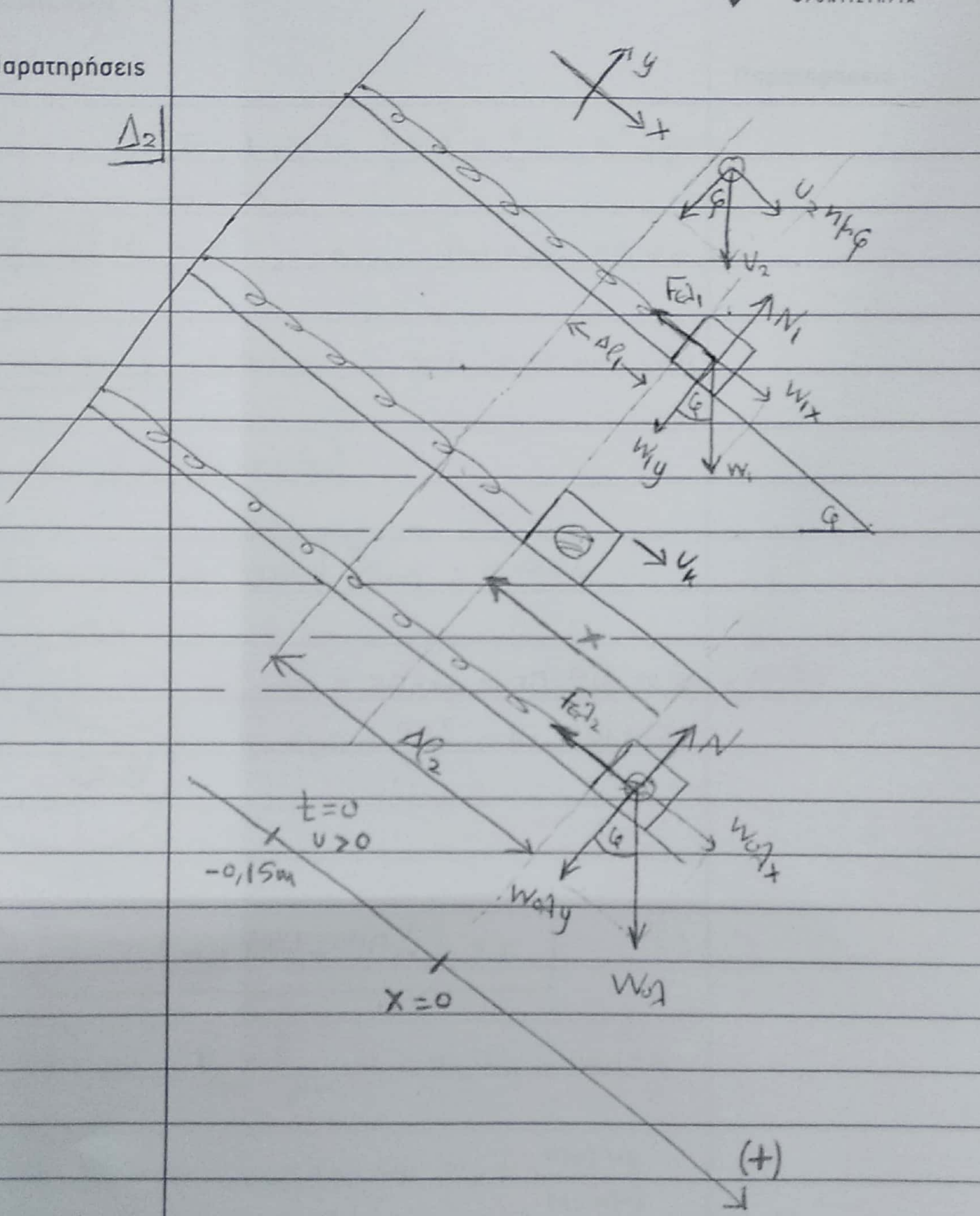
$\Rightarrow N_\Gamma \ell \eta\mu\theta = Mg \frac{\ell}{2} \sigma\upsilon\nu\theta - T_1(\ell/2 + d)\eta\mu\theta$

$\Rightarrow N_\Gamma \ell \eta\mu\theta = Mg \frac{\ell}{2} \sigma\upsilon\nu\theta - T_1(\ell/2 + \ell/6)\eta\mu\theta$

$\Rightarrow N_\Gamma = Mg\frac{1}{2} - T_1 \frac{4}{6} \Rightarrow \boxed{N_\Gamma = 10N}$

Παρατηρήσεις

Δ_2



$Θ.Ι.(m_1)$ $\Sigma F_{1x} = 0 \Rightarrow w_{1x} = F_{ελ_1} \Rightarrow m_1 g\, ημφ = k\Delta l_1$

$$\Rightarrow \Delta l_1 = \frac{m_1 g\, ημφ}{k} = 0{,}05\,m$$

$ΝΕΑ\ Θ.Ι.(m_{ολ})$ $\Sigma F_x = 0 \Rightarrow w_{ολ x} = F_{ελ_2} \Rightarrow m_{ολ} g\, ημφ = k\Delta l_2$

$$\Delta l_2 = \frac{m_{ολ} g\, ημφ}{k} = 0{,}2\,m$$

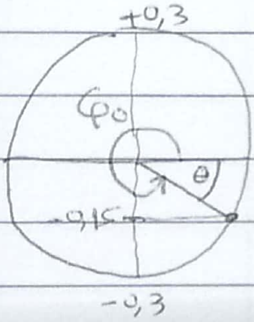
ΑΔΕΤ την $t=0$ $\quad E = K + U \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}m_{ολ}v_k^2 + \frac{1}{2}kx^2$

$A = \sqrt{\frac{m_{ολ}}{k}v_k^2 + x^2}$ $\quad$ όπου $|x| = \Delta\ell_2 - \Delta\ell_1 = 0,15\,m$

$\boxed{A = 0,3\,m}$ $\quad x<0 \rightarrow x = -0,15\,m = -A/2$

Δ3 $\quad x = A\,ημ(ωt + φ_0)$

$D = k = m_{ολ}ω^2 \Rightarrow ω = \sqrt{k/m_{ολ}} = 5\,rad/s$



$φ_0 = 2π - θ = 2π - π/6 \Rightarrow φ_0 = 11π/6$

$ημθ = \frac{0,15}{0,3} = 1/2 \rightarrow θ = π/6$

Άρα $\boxed{x = 0,3\,ημ(5t + 11π/6)\ SI}$

Δ4 $\quad$ ΑΔΟ xx' $\quad \vec{P}_{x\,πριν} = \vec{P}_{x\,μετα} \Rightarrow m_2 v_{2x} = m_{ολ}v_k$

$\Rightarrow m_2 v_2\,ημφ = m_{ολ}v_k \Rightarrow v_2 = \frac{m_{ολ}v_k}{m_2 \cdot ημφ}$

$\Rightarrow \boxed{v_2 = 2\sqrt{3}\ m/s}$

ΘΜΚΕ για m_2: $\quad K_{2τελ} - \cancelto{0}{K_{2αρχ}} = W_{m_2 g}$

$\frac{1}{2}m_2 v_2^2 = m_2 g h$

$h = \frac{v_2^2}{2g} \Rightarrow \boxed{h = 0,6\,m}$

Παρατηρήσεις

Δ5 Θέση μεγίστης επιμήκυνσης

$x = +A \qquad \Delta\ell_{max} = \Delta\ell_2 + A$

$x = 0{,}3m \qquad \Delta\ell_{max} = 0{,}5\,m$

$|\Sigma F| = \Sigma F_{max} = KA = 30N$

$F_{\varepsilon\lambda} = F_{\varepsilon\lambda\,max} = K\Delta\ell_{max} = 50N$

$$\frac{F_{\varepsilon\lambda}}{|\Sigma F|} = \frac{50}{30} \Rightarrow \boxed{\frac{F_{\varepsilon\lambda}}{|\Sigma F|} = 5/3}$$