Παρατηρήσεις

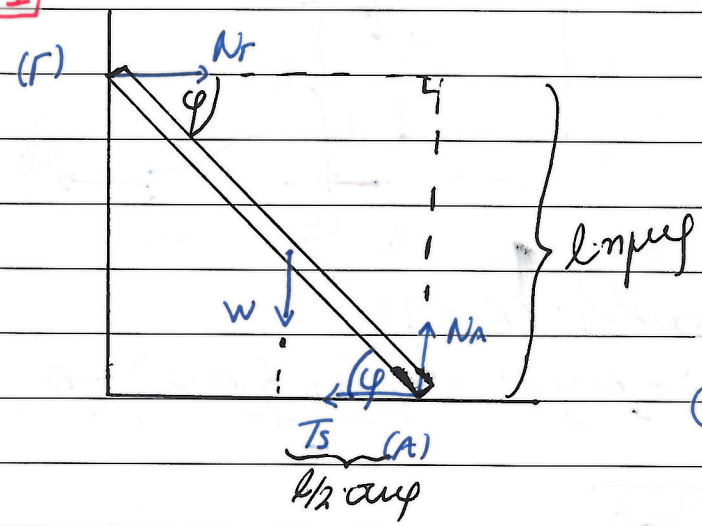
## ΘΕΜΑ Α

**Α1** γ **Α2** δ **Α3** γ **Α4** β

**Α5** α) Σ β) Λ γ) Σ δ) Σ ε) Λ

## ΘΕΜΑ Β

**Β1**




$T_{s\,max} = \mu \cdot N_A$

$\Sigma F_x = 0 \Rightarrow N_r = T_{s\,max}$ (1)

$\Sigma F_y = 0 \Rightarrow N_A = w$ (2)

$\Sigma \tau_{(A)} = 0 \overset{(+)}{\Rightarrow} N_r \cdot l \cdot \eta\mu\varphi = w \frac{l}{2} \cdot \sigma\upsilon\varphi$

$\overset{(1)}{\Rightarrow} T_s \cdot \frac{\eta\mu\varphi}{\sigma\upsilon\varphi} = \frac{w}{2} \Rightarrow T_s = \frac{w}{2\varepsilon\varphi\varphi}$

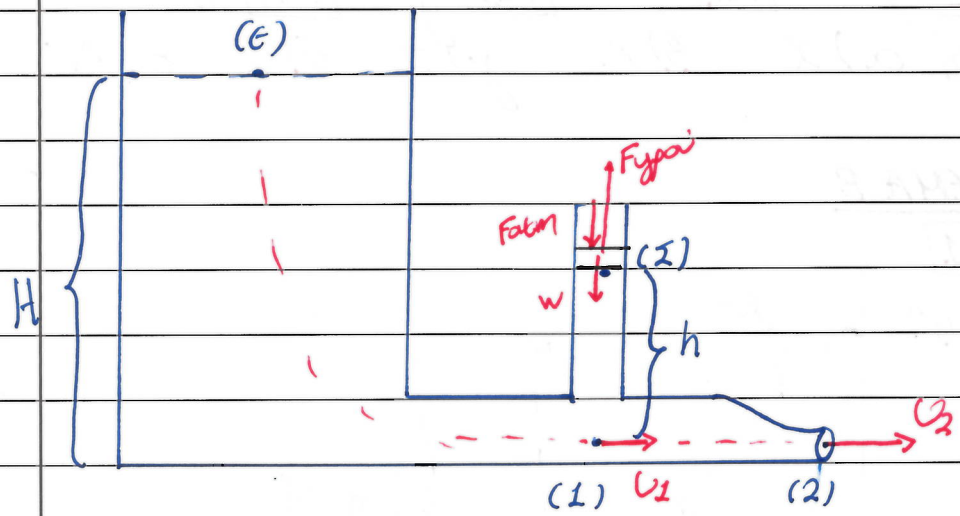
Όμως: $T_s \leq T_{s\,max} \Rightarrow \frac{w}{2\varepsilon\varphi\varphi} \leq \mu \cdot N_A$

$\Rightarrow \frac{w}{2\varepsilon\varphi\varphi} \leq \mu \cdot w \Rightarrow \varepsilon\varphi\varphi \geq \frac{1}{2\mu}$

$\Rightarrow \boxed{\varepsilon\varphi\varphi = \frac{1}{2\mu}}$ (ii)

Παρατηρήσεις

**B2** $A_2 = \frac{A_1}{2}$




Ε.Ι. Bernoulli (Ε → 2)

$P_{atm} + 0 + \rho \cdot g \cdot H = P_{atm} + \frac{1}{2} \cdot \rho \cdot U_2^2 + 0$

$\Rightarrow U_2 = \sqrt{2gH}$

· Εξίσωση συνέχειας:

$\Pi_1 = \Pi_2 \Rightarrow A_1 \cdot U_1 = A_2 \cdot U_2 \Rightarrow A_1 U_1 = \frac{A_1}{2} \cdot U_2$

$\Rightarrow U_1 = \frac{\sqrt{2gH}}{2}$

· Ε.Ι. Bernoulli (1 → 2)

$P_1 + \frac{1}{2} \cdot \rho \cdot U_1^2 = P_{atm} + \frac{1}{2} \cdot \rho U_2^2$

$\Rightarrow P_1 + \frac{1}{2} \cdot \rho \cdot \left(\frac{U_2}{2}\right)^2 = P_{atm} + \frac{1}{2} \cdot \rho U_2^2$

$\Rightarrow P_1 + \frac{1}{2} \cdot \rho \cdot \frac{U_2^2}{4} = P_{atm} + \frac{1}{2} \cdot \rho \cdot U_2^2$

$\Rightarrow P_1 = P_{atm} + \frac{1}{2} \cdot \rho \cdot \frac{3U_2^2}{4}$

Παρατηρήσεις

$\Rightarrow P_1 = P_{atm} + \frac{1}{2}\cdot\rho\cdot\frac{3}{4}\cdot(\sqrt{2gH})^2$

$\Rightarrow P_1 = P_{atm} + \frac{3}{8}\cdot\rho\cdot 2g\cdot H$

$\Rightarrow P_1 = P_{atm} + \frac{3}{4}\cdot\rho\cdot g H \quad (1)$

Ισορροπία εμβόλου:

$\Sigma F = 0 \Rightarrow F_{υγρ} = F_{atm} + W$

$\overset{:A}{\Rightarrow} P_{υγρ} = P_{atm} + \frac{W}{A} \rightarrow P_2$

Άρα:

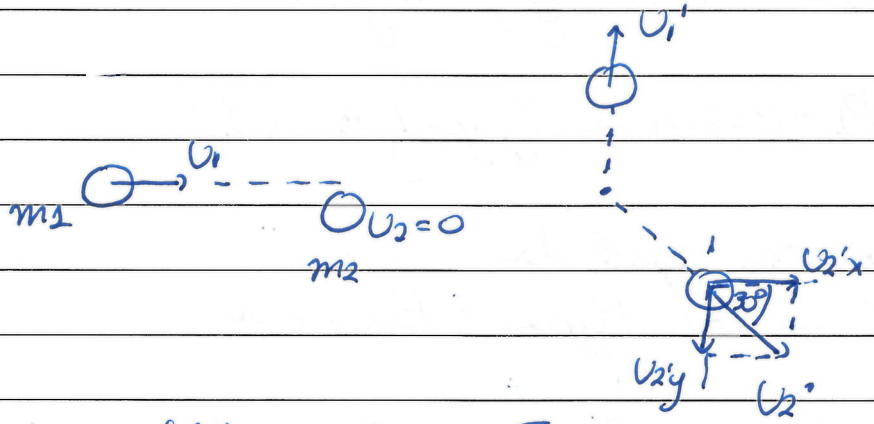
$P_1 = P_2 + \rho\cdot g\cdot h$

$\overset{(1)}{\Rightarrow} P_{atm} + \frac{3}{4}\cdot\rho\cdot g\cdot H = P_{atm} + \frac{W}{A} + \rho\cdot g\cdot h$

$\Rightarrow \frac{3}{4}\cdot\rho\cdot g H - \rho\cdot g\cdot\frac{H}{4} = \frac{W}{A}$

$\Rightarrow \boxed{W = \frac{\rho\cdot g\cdot H\cdot A}{2}}$ (i)

Β3 $m_1 = m$ , $m_2 = 2m$




$U_{2'x} = U_2' \cdot \sigma\upsilon\nu 30° = U_2' \cdot \frac{\sqrt{3}}{2}$

$U_{2'y} = U_2' \cdot \eta\mu 30° = U_2' \cdot \frac{1}{2}$

Α.Δ.Ο. $(x'x)$ $(\xrightarrow{(+)})$

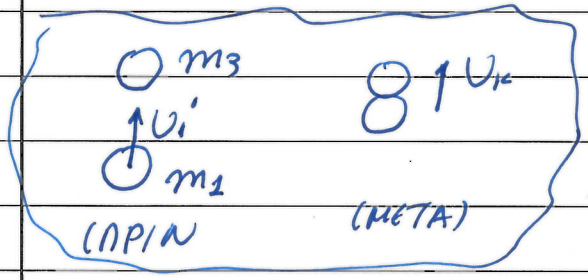
$m_1 \cdot U_1 = 2m_1 \cdot U_{2'x} \Rightarrow U_1 = 2U_2' \cdot \frac{\sqrt{3}}{2}$ (1)

Α.Δ.Ο. $(y'y)$ $(\uparrow +)$

$0 = m_1 \cdot U_1' - 2m_1 \cdot U_{2'y} \Rightarrow U_1' = 2 \cdot U_2' \cdot \frac{1}{2}$

$\Rightarrow U_1' = U_2'$ (2)

(1) $\overset{(2)}{\Rightarrow}$ $U_1 = U_1' \cdot \sqrt{3}$




Α.Δ.Ο. $(\uparrow +)$

$m_1 \cdot U_1' = 2m_1 \cdot U_K$

$\Rightarrow U_K = \frac{U_1'}{2}$

Παρατηρήσεις

- $K_{τελ.} = \frac{1}{2} \cdot 2m \cdot U_k^2 = \frac{1}{2} \cdot 2m \cdot \left(\frac{U_1'}{2}\right)^2$

$\Rightarrow K_{τελ.} = \frac{m \cdot U_1'^2}{4}$

- $K_{αρχ} = \frac{1}{2} \cdot m \cdot U_1^2 = \frac{1}{2} \cdot m \cdot (U_1' \cdot \sqrt{3})^2$

$= \frac{1}{2} \cdot m \cdot U_1'^2 \cdot 3 = \frac{3}{2} m \cdot U_1'^2$

$$\frac{K_{τελ}}{K_{αρχ}} = \frac{\frac{m \cdot U_1'^2}{4}}{\frac{3m \cdot U_1'^2}{2}} = \frac{1}{6}$$

## ΘΕΜΑ Γ

$\ell = 1\,m$

$m = 0,5\,kg$

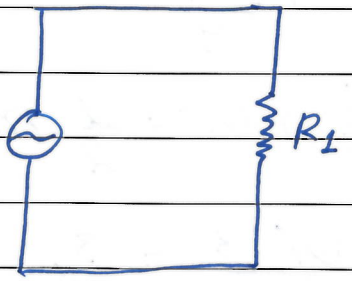
$R_{κλ} = 2\,\Omega$

$U = V \cdot ημ(50\pi t)$

$R_1 = 6\,\Omega$

$R_2 = 3\,\Omega$

**Γ1** $V = ;$, $I_{ενεργ} = ;$




$P_{R_1} = I_{ενεργ}^2 \cdot R_1 \Rightarrow 12 = I_{ενεργ}^2 \cdot 6 \Rightarrow I_{ενεργ}^2 = 2$

$\Rightarrow \boxed{I_{ενεργ} = \sqrt{2}\,A}$

$I_{ενεργ} = \dfrac{I}{\sqrt{2}} \Rightarrow I = 2\,A$

$I = \dfrac{V}{R_1} \Rightarrow \boxed{V = 12\,V}$

**Γ2** $\rho_{R_1} = ;$, $t = 5 \cdot 10^{-3}\,s$

$\left.\begin{array}{l} \omega' = 2\omega \rightarrow V' = N \cdot \omega' \cdot B \cdot A \\ V = N \cdot \omega \cdot B \cdot A \end{array}\right\} \overset{:}{\Rightarrow} \dfrac{V'}{V} = \dfrac{2\omega}{\omega}$

$\Rightarrow V' = 24\,V$

$I' = \dfrac{V'}{R_1} = \dfrac{24}{6} = 4\,A$

$\rho_{R_1} = U \cdot i = V' \cdot ημ(\omega t) \cdot I' \cdot ημ(\omega t)$

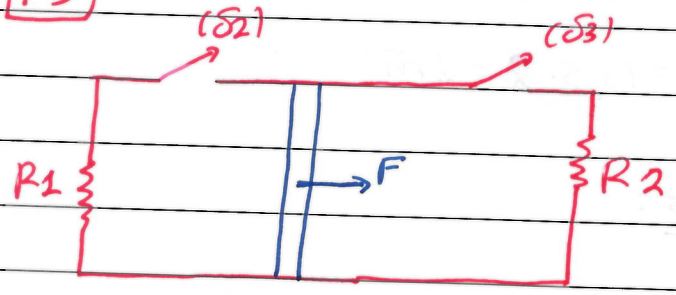
$\Rightarrow \boxed{\rho_{R_1} = 96 \cdot ημ^2(100\pi t) \quad (S.I.)}$

Παρατηρήσεις

$P_{R_1} = 96 \cdot \eta\mu^2(100\pi \cdot 5 \cdot 10^{-3})$

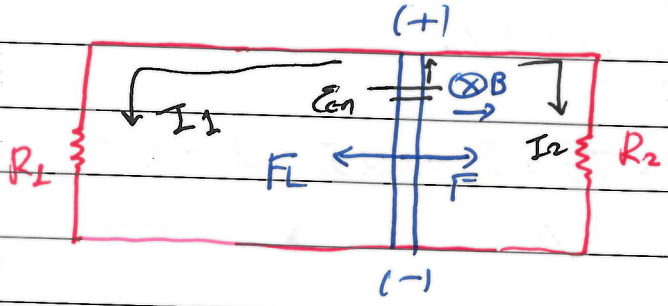
$\Rightarrow P_{R_1} = 96 \cdot \eta\mu^2\left(\frac{\pi}{2}\right) \Rightarrow \boxed{P_{R_1} = 96\,W}$

**Γ3**




Από 0-2s : $F = m \cdot a \Rightarrow a = \frac{0,5}{0,5} = 1\,m/s^2$

$U = a \cdot t = 2\,m/s$




$R_{ΕΞ} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 2\,\Omega$ , $R_{ολ} = R_{ΕΞ} + R_{εσ} = 4\,\Omega$

$\Sigma F = 0 \Rightarrow F_L = F \Rightarrow B \cdot I \cdot \ell = F \Rightarrow B \cdot \frac{\mathcal{E}_{επ}}{R_{ολ}} \cdot \ell = F$

$\Rightarrow B \cdot \frac{B \cdot U_{op} \ell}{R_{ολ}} \cdot \ell = F \Rightarrow B^2 = \frac{F \cdot R_{ολ}}{U_{op} \cdot \ell^2} = \frac{0,5 \cdot 4}{2 \cdot 1}$

$\Rightarrow \boxed{B = 1\,T}$

[Γ4] $\pi = \frac{Q_{R_2}}{W_F} \cdot 100\% = ;$

• Από 0-2s : α=σταθερή

$\Delta x_1 = \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} 1 \cdot 2^2 = 2m$

$W_F = F \cdot \Delta x_1 = 0,5 \cdot 2 = 1\ J$

• Από 2-5s :

$V_1 = V_2 \Rightarrow I_1 \cdot R_1 = I_2 \cdot R_2 \Rightarrow I_1 \cdot 6 = I_2 \cdot 3$

$\Rightarrow I_2 = 2 \cdot I_1 \ (1)$

$\mathcal{E}_{επ} = B \cdot v \cdot \ell = 1 \cdot 2 \cdot 1 = 2V$

$I = \frac{\mathcal{E}_{επ}}{R_{ολ}} = \frac{2}{4} = 0,5\ A$

Όμως: $I_1 + I_2 = I \Rightarrow 2I_1 + I_1 = 0,5$

$\Rightarrow I_1 = \frac{0,5}{3}\ A$

$(1) \Rightarrow I_2 = \frac{1}{3}\ A$

$Q_{R_2} = I_2^2 \cdot R_2 \cdot \Delta t = \frac{1}{9} \cdot 3 \cdot (5-2) = 1\ J$

• $\Delta x_2 = v \cdot \Delta t = 2 \cdot (5-2) = 6m$

$W_F' = F \cdot \Delta x_2 = 3\ J$

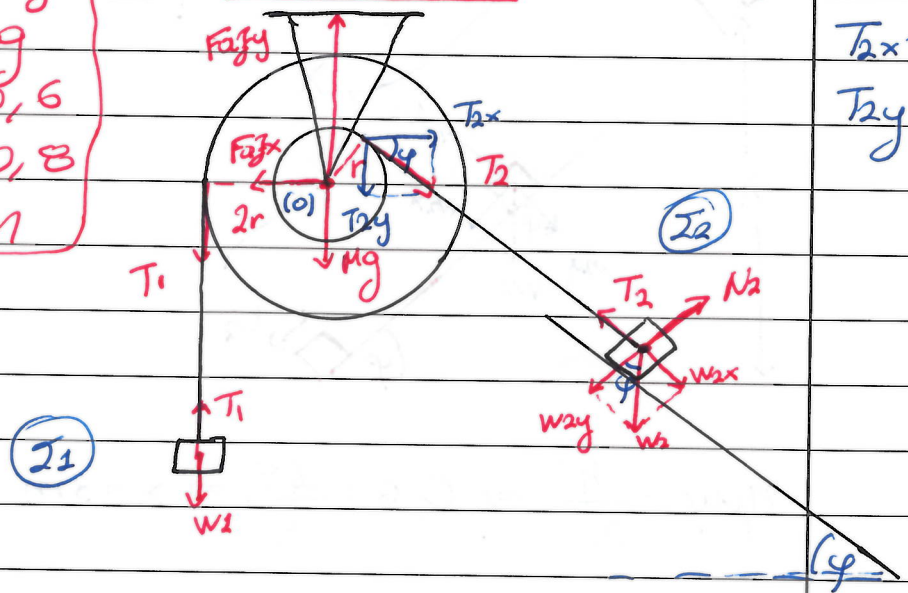
$W_{F_{ολ}} = W_F + W_F' = 4\ J$

Άρα: $\boxed{\pi = \frac{1}{4} \cdot 100\% = 25\%}$

# ΘΕΜΑ Δ

$M = 1,5\,kg$
$m_2 = 5\,kg$
$\eta\mu\varphi = 0,6$
$\sigma\upsilon\nu\varphi = 0,8$
$d = 0,2\,m$

**Δ1** $m_1 = ?$, $F_{αJ} = ?$




Παρατηρήσεις

$T_{2x} = T_2 \cdot \sigma\upsilon\nu\varphi$
$T_{2y} = T_2 \cdot \eta\mu\varphi$

• **Σ2**

$\Sigma F_{2x} = 0 \Rightarrow T_2 = w_{2x} = m_2 g \cdot \eta\mu\varphi = 30N$

$\Sigma \tau_{(O)} = 0 \overset{+}{\circlearrowleft} \Rightarrow T_1 \cdot 2r = T_2 \cdot r \Rightarrow T_1 = 15N$

• **Σ1**

$\Sigma F_{1y} = 0 \Rightarrow T_1 = m_1 \cdot g \Rightarrow \boxed{m_1 = 1,5\,kg}$

• **Τροχαλία**

$\Sigma F_x = 0 \Rightarrow F_{αJx} = T_2 \cdot \sigma\upsilon\nu\varphi \Rightarrow F_{αJx} = 24N$

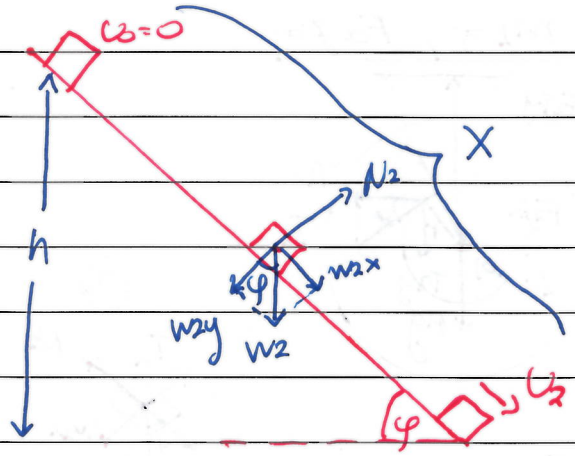
$\Sigma F_y = 0 \Rightarrow F_{αJy} = T_1 + T_{2y} + Mg = 15 + 18 + 15$

$\Rightarrow F_{αJy} = 48N = 2 \cdot 24\,N$

$F_{αJ} = \sqrt{F_{αJx}^2 + F_{αJy}^2} = \sqrt{24^2 + 4 \cdot 24^2} = 24\sqrt{5}N$

Παρατηρήσεις

**Δ2** $h = 1,8m$



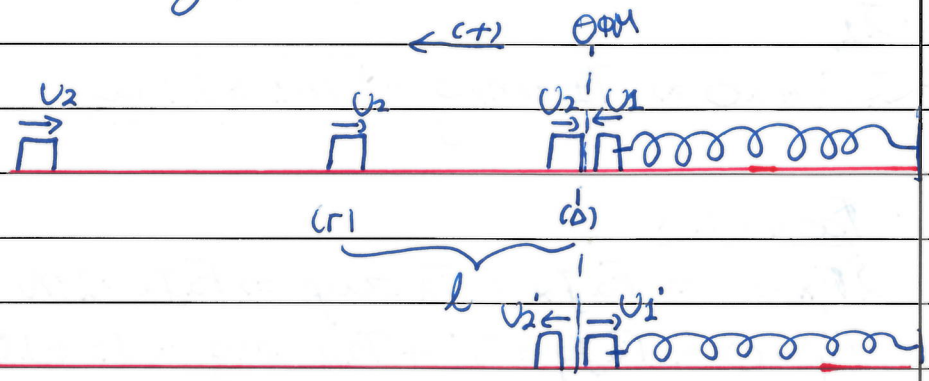

$$\eta\mu\varphi = \frac{h}{x} \Rightarrow 0,6 = \frac{1,8}{x} \Rightarrow x = 3m$$

$$W_{2x} = W_2 \cdot \eta\mu\varphi = 30N$$

Θ.Μ.Κ.Ε.

$$\frac{1}{2} \cdot m_2 \upsilon_2^2 - 0 = W_{2x} \cdot x \Rightarrow \frac{1}{2} m_2 \cdot \upsilon_2^2 = m_2 g \cdot \eta\mu\varphi \, x$$

$$\Rightarrow \upsilon_2 = \sqrt{2g\eta\mu\varphi x} = \sqrt{20 \cdot 0,6 \cdot 3} = 6 m/s$$




Το Σ3 κινείται για χρόνο $\frac{T}{4}$ πριν την ισορροπία.

Το Σ3 κινείται για χρόνο:

$$\upsilon_2 = \frac{l}{\Delta t} \Rightarrow \Delta t = \frac{l}{\upsilon_2} = \frac{3\pi/5}{6} \Rightarrow \Delta t = \frac{\pi}{10} s$$

Παρατηρήσεις

Άρα: $\Delta t = \frac{T}{4} = \frac{\pi}{10} \Rightarrow T = 0,4\pi\, s$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0,4\pi} \Rightarrow \omega = 5\, rad/s$

$K = m_3 \cdot \omega^2 = 5 \cdot 5^2 \Rightarrow \boxed{K = 125\, N/m}$

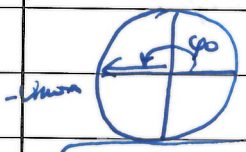
**Δ3** $x = f(t)$

$\rightsquigarrow U_1 = U_{max} = \omega \cdot A = 5 \cdot 0,2 = 1\, m/s$

$\cdot\cdot\cdot$

$\cdot\cdot$ $m_1 = m_2 \longrightarrow U_1' = U_2 = 6\, m/s$

Άρα: $U_1' = \omega A' \Rightarrow A' = \frac{6}{5} = 1,2\, m$




$\varphi_0 = \pi\, rad$

$\boxed{x = 1,2 \cdot \eta\mu(5t + \pi)\ (S.I.)}$

**Δ4** $K = 8U$, $\frac{dp_3}{dt} = ;$, $\left|\frac{dK}{dt}\right| = ;$

Α.Δ.Ε.Τ: $E = K + U \Rightarrow E = 9U$

$\Rightarrow \frac{1}{2} D A^2 = 9 \cdot \frac{1}{2} D \cdot x^2 \Rightarrow x = \pm \frac{A}{3}$

· Για 1η φορά: $x = -\frac{A}{3}$

$\frac{dp_3}{dt} = -D \cdot x = -125 \cdot \left(-\frac{1,2}{3}\right) \Rightarrow \boxed{\frac{dp_3}{dt} = +50\, N}$

· $K = 8U \Rightarrow \frac{1}{2} m_3 \cdot U^2 = 8 \cdot \frac{1}{2} \cdot D \cdot x^2$

Παρατηρήσεις

$\cancel{m_3} \cdot U^2 = \Theta \cancel{m_3} \cdot \omega^2 \cdot x^2 \Rightarrow |U| = 2\sqrt{2} \cdot \omega \cdot x$

$\Rightarrow |U| = 2\sqrt{2} \cdot \omega \frac{A}{3}$

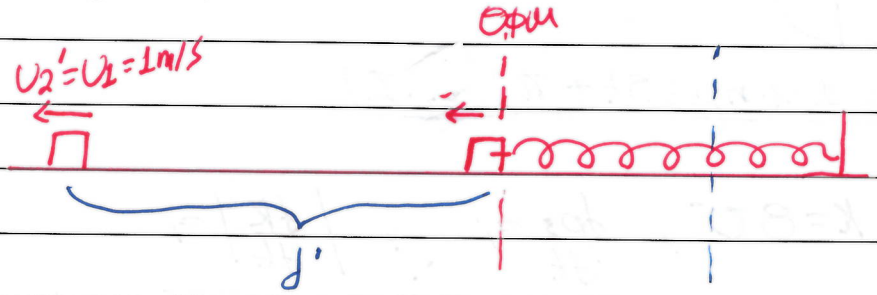
$\Rightarrow |U| = 2 \cdot \sqrt{2} \cdot 5 \cdot \frac{1,2}{3} \Rightarrow |U| = 4\sqrt{2}\ m/s$

$\cdot \left|\frac{dK}{dt}\right| = |\Sigma F \cdot U| = |-D \cdot x \cdot U| = 125 \cdot \frac{1,2}{3} \cdot 4\sqrt{2}$

$\Rightarrow \boxed{\left|\frac{dK}{dt}\right| = 200\sqrt{2}\ J/s}$

Δ5) $d' = ;$

Το $\Sigma_3$ κινείται για χρόνο $\frac{T}{2}$ μέχρι να ξαναβρεθεί στην ίδια θέση.




Στον ίδιο χρόνο, το σώμα $\Sigma_2$ έχει διανύσει:

$d' = U_2' \cdot \frac{T}{2} = 1 \cdot \frac{0,4\pi}{2} \Rightarrow \boxed{d' = 0,2\pi\ m}$

$\rightarrow \boxed{d' = 0,628 m}$