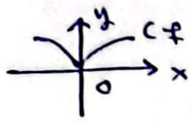
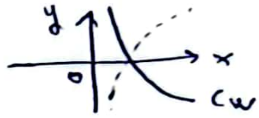
ΛΥΣΕΙΣ ΔΙΑΓ. Β. ΛΥΚΕΙΟΥ ΜΑΘΗΜΑΤΙΚΑ

3/3/2024

ΘΕΜΑ Α

A3 α) 

β) 

γ) 

A4 α) Λ β) Σ γ) Λ δ) Σ ε) Σ

ΘΕΜΑ Β

B1 $A_f = (-6,1)\cup(1,5]$, $f(A) = [-1,5]$

B2 $f(x) = 0 \Leftrightarrow x=-5$ ή $x=-3$ ή $x=5$

B3 $f(x) < 0$ ΟΤΑΝ $x \in (-6,-5)\cup\{3\}$

B4 $\lim\limits_{x\to -6} f(x) = -1$, $\lim\limits_{x\to -2} f(x) \nexists$, $\lim\limits_{x\to 1} f(x) = 3$, $\lim\limits_{x\to 3} f(x) = 2$, $\lim\limits_{x\to 4} f(x) \nexists$

B5 3 ακριβώς ρίζες

ΘΕΜΑ Γ

Γ1 • $f(-1)=0 \Leftrightarrow 1-\alpha+\beta+\cancel{16}-\cancel{12}=0 \Leftrightarrow \boxed{-\alpha+\beta=-5}$ (1)

• $f(1)=-32 \Leftrightarrow 1+\alpha+\beta-16-\cancel{12}=-24 \Leftrightarrow \boxed{\alpha+\beta=3}$ (1)

$(1)+(2) \Rightarrow 2\beta=-2 \Leftrightarrow \boxed{\beta=-1}$ άρα $\boxed{\alpha=4}$

Γ2 Είναι $f(x) = x^4+4x^3-x^2-16x-12$

1	4	−1	−16	−12	−1
↓	−1	−3	4	12	
1	3	−4	−12	0	

άρα $f(x) = (x+1)(x^3+3x^2-4x-12)$

• $2x^2+x-1 = 2(x+1)(x-\frac{1}{2})$

$\Delta = 9$, $x = \dfrac{-1+3}{4} = \dfrac{2}{4} = 1/2$, $x = \dfrac{-1-3}{4} = \dfrac{-4}{4} = -1$

• $\lim\limits_{x\to -1} \dfrac{f(x)}{2x^2+x-1} \overset{\frac{0}{0}}{=} \lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}(x^3+3x^2-4x-12)}{2\cancel{(x+1)}(x-\frac{1}{2})} = \dfrac{-1+3+4-12}{2(-1-\frac{1}{2})} = \boxed{2}$

Γ3

$f(x) = (x+1)(x^3+3x^2-4x-12)$

1	3	-4	-12	2
↓	2	10	12	
1	5	6	0	

άρα $f(x) = (x+1)(x-2)(x^2+5x+6)$

$x=-1$, $x=2$, $\Delta = 25-24 = 1$

$x = \frac{-5-1}{2} = \frac{-6}{2} = -3$ ή $x = \frac{-5+1}{2} = \frac{-4}{2} = -2$

x	$-\infty$	-3		-2		-1		2	$+\infty$
$x+1$	$-$		$-$		$-$	0	$+$		$+$
$x-2$	$-$		$-$		$-$		$-$	0	$+$
x^2+5x+6	$+$	0	$-$	0	$+$		$+$		$+$
$f(x)$	$+$	0	$-$	0	$+$	0	$-$	0	$+$

άρα $f(x)>0$ όταν

$x \in (-\infty,-3) \cup (-2,-1) \cup (2,+\infty)$

Γ4

$\frac{f(x)}{x^2+5x+6} \geq 0 \Leftrightarrow$ ΠΕΡΙΟΡ.: $x^2+5x+6 \neq 0 \rightarrow \boxed{x \neq -3}$ κ' $\boxed{x \neq -2}$

$\Leftrightarrow \frac{(x+1)(x-2)\cancel{(x^2+5x+6)}}{\cancel{x^2+5x+6}} \geq 0 \Leftrightarrow (x+1)(x-2) \geq 0$

x		-3		-2		-1		2	
$x+1$	$-$	‖	$-$	‖	$-$	0	$+$		$+$
$x-2$	$-$	‖	$-$	‖	$-$		$-$	0	$+$
Γιν.	$+$	‖	$+$	‖	$+$	0	$-$	0	$+$

οπότε,
$x \in (-\infty,-3) \cup (-3,-2) \cup (-2,-1] \cup [2,+\infty)$

ΘΕΜΑ Δ

Δ1 ▷ ΠΕΡΙΟΡ.: $x \neq 0$ ΚΑΙ $\frac{e^x-1}{x} > 0 \Leftrightarrow (e^x-1)\cdot x > 0$

x		0	
x	$-$	0	$+$
e^x-1	$-$	0	$+$
Γιν	$+$	‖	$+$

άρα $A_f = \mathbb{R} - \{0\}$

▷ ΠΕΡΙΟΡ.: $x>0$ ΚΑΙ $e^{x^2}-1>0 \Leftrightarrow e^{x^2}>1 \Leftrightarrow x^2>0 \Rightarrow x \neq 0$

άρα $A_g = (0,+\infty)$

$\boxed{\Delta 2}$

$$f(2x) - f(x) = \ln(e^{2x} - 2e^x + 3) - \ln 2 \quad , \quad x \neq 0$$

$$\ln\left(\frac{e^{2x}-1}{2x}\right) - \ln\left(\frac{e^x-1}{x}\right) = \ln\left(\frac{e^{2x}-2e^x+3}{2}\right) \Leftrightarrow$$

$$\ln\left(\frac{\frac{e^{2x}-1}{2x}}{\frac{e^x-1}{\cancel{x}}}\right) = \ln\left(\frac{e^{2x}-2e^x+3}{2}\right) \Leftrightarrow$$

$$\ln\left(\frac{e^{2x}-1}{2(e^x-1)}\right) = \ln\left(\frac{e^{2x}-2e^x+3}{2}\right) \Leftrightarrow \frac{e^{2x}-1}{2(e^x-1)} = \frac{e^{2x}-2e^x+3}{2}$$

$$\Leftrightarrow \frac{\cancel{(e^x-1)}(e^x+1)}{\cancel{e^x-1}} = e^{2x} - 2e^x + 3 \Leftrightarrow e^x + 1 = e^{2x} - 2e^x + 3$$

$$\Leftrightarrow e^{2x} - 3e^x + 2 = 0 \qquad \Delta = 9 - 8 = 1$$

$$e^x = \begin{cases} \frac{3+1}{2} = 2 \to e^x = 2 \Leftrightarrow \boxed{x = \ln 2} \text{ ΔΕΚΤΗ} \\ \frac{3-1}{2} = 1 \to e^x = 1 \Leftrightarrow \boxed{x = 0} \text{ Απορ.} \end{cases}$$

$\boxed{\Delta 3}$ $f(x) + \ln x > g(x) + 2\ln x \quad , \quad \boxed{x > 0}$

$$\ln\left(\frac{e^x-1}{x}\right) + \ln x > \ln(e^{x^2}-1) - \cancel{2\ln x} + \cancel{2\ln x} \Leftrightarrow$$

$$\ln(e^x - 1) > \ln(e^{x^2} - 1) \Leftrightarrow e^x - \cancel{1} > e^{x^2} - \cancel{1} \Leftrightarrow x > x^2$$

$:x>0$

$\Leftrightarrow \quad 1 > x \Leftrightarrow x < 1 \quad$ οπότε $\boxed{0 < x < 1}$

$\boxed{\Delta 4}$ α) $f(-x) = \ln\left(\frac{e^{-x}-1}{-x}\right) = \ln\left(\frac{\frac{1-e^x}{e^x}}{-x}\right) = \ln\left(\frac{e^x-1}{x} \cdot \frac{1}{e^x}\right) =$

$$= \ln\left(\frac{e^x-1}{x}\right) + \ln\frac{1}{e^x} = f(x) - x$$

άρα $f(-x) = f(x) - x \Leftrightarrow f(x) = f(-x) + x$

β) Για $x = 2024$: $f(2024) = f(-2024) + 2024 \Leftrightarrow$

$$f(2024) - f(-2024) = 2024 > 0 \Leftrightarrow f(2024) > f(-2024)$$