

ΘΕΜΑ Α

A3 1. Λ 2. Λ 3. Λ 4. Σ 5. Λ

ΘΕΜΑ Β

B1 α) ΠΡΕΣΗ: $x+2 \neq 0$ κ' $x-3 \neq 0$ $\Leftrightarrow x \in \mathbb{R} - \{-2, 3\}$
 $x \neq -2$ κ' $x \neq 3$

β) $K = \frac{|x+2|}{x+2} - \frac{|x-3|}{x-3} = \frac{x+2}{x+2} + \frac{x-3}{x-3} = 1+1=2$

B2 $A = 2^{\frac{6}{2}} = 2^3 = 8$, $B = 2^{\frac{6}{3}} = 2^2 = 4$

α) $A-B=4 \Leftrightarrow 8-4=4$ β) $1 < \sqrt[3]{2} < \sqrt{2}$

ΘΕΜΑ Γ

α) ΑΔΥΝΑΤΗ β) $x \neq -1$ $|x+2| = |x+1| \Leftrightarrow x+2 = x+1$ ή $x+2 = -(x+1)$
 $2=1$ ή $2x = -3$
 $x = -\frac{3}{2}$

γ) $|x-1| + |x^2-x| = 0 \Leftrightarrow x=1$ και $x^2-x=0 \Leftrightarrow x=0$ ή $x=1$
 αρα $\boxed{x=1}$

δ) Αν $x+7 \geq 0 \Leftrightarrow x \geq -7$: $x+7 = 2x-1 \Leftrightarrow -x = -8 \Leftrightarrow \boxed{x=8}$

Αν $x+7 < 0 \Leftrightarrow x < -7$: $-x-7 = 2x-1 \Leftrightarrow -3x = 6 \Leftrightarrow \boxed{x=-2}$

ε) $\frac{2-|x-2|}{3} - \frac{1-2|x-2|}{2} = |x-2| - \frac{8-|x-2|}{6} \Leftrightarrow$

$\frac{2-y}{3} - \frac{1-2y}{2} = y - \frac{8-y}{6} \Leftrightarrow 2(2-y) - 3(1-2y) = 6y - 8 + y$

$\Leftrightarrow 4-2y-3+6y = 6y-8+y \Leftrightarrow -3y+1 = -8 \Leftrightarrow -3y = -9$

$\Leftrightarrow y=3 \Leftrightarrow |x-2|=3 \Leftrightarrow x-2=3$ ή $x-2=-3$

$\Leftrightarrow \boxed{x=5}$ ή $\boxed{x=-1}$

ΘΕΜΑ Δ

$$\Delta 1 \quad \alpha^2 = (1+\sqrt{2})^2 = 1 + 2\sqrt{2} + 2 = 3 + 2\sqrt{2}$$
$$\beta^2 = (1-\sqrt{2})^2 = 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$$

$$\Delta 2 \quad \bullet A = \alpha^2 - \beta^2 = 3 + 2\sqrt{2} - 3 + 2\sqrt{2} = 4\sqrt{2}$$
$$\bullet B = \sqrt{(1+\sqrt{2})^2} - \sqrt{(1-\sqrt{2})^2} = |1+\sqrt{2}| - |1-\sqrt{2}| = 1+\sqrt{2} + 1-\sqrt{2} = 2$$
$$\bullet \Gamma = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{3+2\sqrt{2}} + \frac{1}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} + \frac{3+2\sqrt{2}}{9-8} = 6$$

$$\Delta 3 \quad \text{Εστω, } \sqrt{A} > |\alpha| - |\beta| \Leftrightarrow \sqrt{A} > B \Leftrightarrow \sqrt{4\sqrt{2}} > 2$$
$$\Leftrightarrow 2\sqrt{2} > 4 \Leftrightarrow \sqrt{2} > 1 \quad \text{ΙΣΧΥΗ}$$

$$\Delta 4 \quad \text{Εστω } \left|x + \frac{1}{x}\right| \geq 2 \Leftrightarrow \left|\frac{x^2+1}{x}\right| \geq 2 \Leftrightarrow \frac{x^2+1}{|x|} \geq 2 \Leftrightarrow x^2+1 \geq 2|x|$$
$$\Leftrightarrow |x|^2 - 2|x| + 1 \geq 0 \Leftrightarrow (|x|-1)^2 \geq 0 \quad \text{ΙΣΧΥΗ}$$

ΤΟ ΙΣΟΝ ΙΣΧΥΕΙ ΜΟΝΟ ΟΤΑΝ $|x|=1 \Leftrightarrow x=1 \text{ ή } x=-1$

$$\Delta 5 \quad \text{Εξοχμα} \quad \sqrt[2024]{1-x} + \left|x + \frac{1}{x}\right| - 2 = 0$$

$$\text{Επιπλέον} \quad \sqrt[2024]{1-x} \geq 0 \quad \text{κ' } \left|x + \frac{1}{x}\right| - 2 \geq 0$$

$$\text{Τότε} \quad \sqrt[2024]{1-x} = 0 \quad \text{και} \quad \left|x + \frac{1}{x}\right| - 2 = 0$$

$$1-x=0$$

$$x=1$$

$$\text{Από } \Delta 4 \quad x=1 \text{ ή } x=-1$$

$$\text{Άρα } x=1$$