Εν Δυνάμει
ΦΡΟΝΤΙΣΤΗΡΙΑ

Παρατηρήσεις

ΜΑΘΗΜΑΤΙΚΑ Β' ΛΥΚΕΙΟΥ

12-11-23

ΘΕΜΑ Α

Α2) i) Λ ii) Λ iii) Δ iv) Λ v) Σ
vi) Λ vii) Λ

Α3) Με σημείο αναφοράς το Κ

$2\vec{KA} - 2\vec{KA} + 3\vec{KA} - 3\vec{KB} + 2\vec{KB} - 2\vec{KM} = -\vec{KA} + \vec{KM} - \vec{KA} - \vec{KB} \Leftrightarrow$

$5\vec{KA} = 3\vec{KM} \Leftrightarrow \vec{KA} = \frac{3}{5}\vec{KM}$ και $\vec{KA} \parallel \vec{KM}$ } $\Rightarrow$ Κ, Λ, Μ συνευθειακά
Κ κοινό }

ΘΕΜΑ Β

Β1) $\vec{\Delta E} = \vec{AE} - \vec{A\Delta} = \frac{5}{6}\vec{AB} + \frac{2}{3}\vec{A\Gamma} - \frac{2}{3}\vec{AB} - \frac{5}{6}\vec{A\Gamma} = \frac{1}{6}\vec{AB} - \frac{1}{6}\vec{A\Gamma} = \frac{1}{6}(\vec{AB} - \vec{A\Gamma}) =$
$\frac{1}{6}\vec{\Gamma B} = -\frac{1}{6}\vec{B\Gamma}$ άρα $\vec{\Delta E} \parallel \vec{B\Gamma}$

Β2) i) $\vec{\alpha}\cdot\vec{\beta} = |\vec{\alpha}||\vec{\beta}|\sigma\upsilon\nu(\hat{\vec{\alpha},\vec{\beta}}) = \sqrt{2}\cdot 2\sqrt{2}\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$

ii) $\vec{\alpha}^2 + \vec{\beta}^2 = |\vec{\alpha}|^2 + |\vec{\beta}|^2 = 2 + 8 = 10$

iii) $(\vec{\alpha}+\vec{\beta})^2 = \vec{\alpha}^2 + 2\vec{\alpha}\vec{\beta} + \vec{\beta}^2 = 10 + 4\sqrt{3}$

iv) $|\vec{\alpha}+\vec{\beta}|^2 = (\vec{\alpha}+\vec{\beta})^2 = 10 + 4\sqrt{3}$ άρα $|\vec{\alpha}+\vec{\beta}| = \sqrt{10+4\sqrt{3}}$

v) $(2\vec{\alpha}+3\vec{\beta})(4\vec{\alpha}-5\vec{\beta}) = 8\vec{\alpha}^2 - 10\vec{\alpha}\vec{\beta} + 12\vec{\alpha}\vec{\beta} - 15\vec{\beta}^2 = 8|\vec{\alpha}|^2 + 2\vec{\alpha}\vec{\beta} - 15|\vec{\beta}|^2 =$
$= 16 + 4\sqrt{3} - 120 = 4\sqrt{3} - 104$

Β3) i) $|\vec{\alpha}| = \sqrt{(|\vec{\alpha}|-2)^2 + 4^2} \Leftrightarrow |\vec{\alpha}|^2 = |\vec{\alpha}|^2 - 4|\vec{\alpha}| + 4 + 16 \Leftrightarrow 4|\vec{\alpha}| = 20 \Leftrightarrow |\vec{\alpha}| = 5$

ii) $\vec{\alpha} = (3,4)$ άρα $\vec{u} = (x-3, y-4)$ με $\lambda_{\vec{u}} = \varepsilon\varphi 225 \Leftrightarrow \lambda_{\vec{u}} = \varepsilon\varphi(180+45) \Leftrightarrow$
$\lambda_{\vec{u}} = \varepsilon\varphi 45° \Leftrightarrow \lambda_{\vec{u}} = 1 \Leftrightarrow \frac{y-4}{x-3} = 1 \Leftrightarrow y-4 = x-3 \Leftrightarrow \boxed{y-x=1}$

$|\vec{\beta}| = \sqrt{5} \Leftrightarrow \sqrt{x^2+y^2} = \sqrt{5} \Leftrightarrow \boxed{x^2+y^2=5}$

Παρατηρήσεις

$\begin{cases} x^2+y^2=5 \\ y-x=1 \Leftrightarrow y=x+1 \end{cases} \Rightarrow x^2+(x+1)^2-5=0 \Leftrightarrow x^2+x^2+2x+1-5=0 \Leftrightarrow$

$2x^2+2x-4=0 \Leftrightarrow x^2+x-2=0 \rightarrow x=1$, $x=-2$ δεκτά

$\vec{\beta}=(1,2)$ ή $\vec{\beta}=(-2,-1)$

iii) $\vec{\beta}=(-2,-1)$, $\det(\vec{\alpha},\vec{\beta})=\begin{vmatrix} 3 & 4 \\ -2 & -1 \end{vmatrix}=-3+8=5\neq 0$ δεκτά

$\vec{\alpha}\not\parallel\vec{\beta}$

iv) Έστω $\vec{\gamma}$ το ζητούμενο διάνυσμα

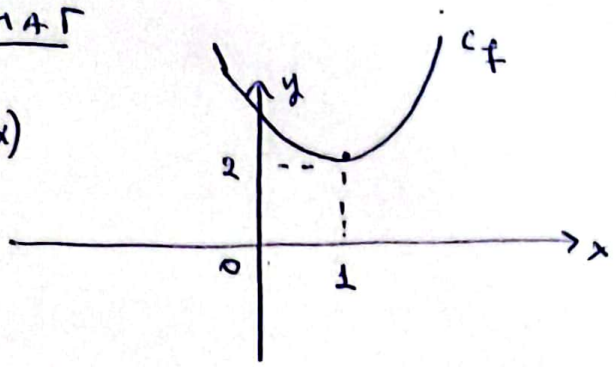
$\vec{\gamma}\uparrow\downarrow\vec{\alpha} \Leftrightarrow \vec{\gamma}=\lambda\vec{\alpha}$ με $\lambda<0$

$|\vec{\gamma}|=3|\vec{\alpha}| \Leftrightarrow |\lambda\vec{\alpha}|=3|\vec{\alpha}| \Leftrightarrow |\lambda||\vec{\alpha}|=3|\vec{\alpha}| \Leftrightarrow |\lambda|=3 \overset{\lambda<0}{\Leftrightarrow} \lambda=-3$

δεκτό $\vec{\gamma}=-3\vec{\alpha} \Leftrightarrow \vec{\gamma}=(-9,-12)$

ΘΕΜΑ Γ

Γ3 α)



$f \downarrow$ στο $(-\infty, 1]$

$f \uparrow$ στο $[1, +\infty)$

$ΟΕ = f(1) = 2 = f_{min}$

β) $g(x) = f(x+1) - 4 = (x-1+1)^2 + 2 - 4 = x^2 - 2$

$g(-x) = (-x)^2 - 2 = g(x)$ άρα g ΑΡΤΙΑ

Γ4 1. Λ 2. Λ 3. Σ 4. Λ 5. Λ

ΘΕΜΑ Δ

Δ1 α) $4\eta\mu^2\theta + 4\eta\mu\theta + 1 = 0 \Leftrightarrow (2\eta\mu\theta + 1)^2 = 0 \Leftrightarrow 2\eta\mu\theta + 1 = 0 \Leftrightarrow \eta\mu\theta = -\frac{1}{2}$

β) $\varepsilon\varphi\left(\frac{15\pi}{2} + \theta\right) = \varepsilon\varphi\left(\frac{3\pi}{2} + \theta\right) = -\varepsilon\varphi\theta$ | $\varepsilon\varphi\left(\frac{17\pi}{2} - \theta\right) = \varepsilon\varphi\left(\frac{\pi}{2} - \theta\right) = \sigma\varphi\theta$

$\sigma\upsilon\nu(\theta - 10\pi) = \sigma\upsilon\nu\theta$ | $\sigma\upsilon\nu\left(\frac{11\pi}{2} + \theta\right) = \sigma\upsilon\nu\left(\frac{3\pi}{2} + \theta\right) = \eta\mu\theta$

$$A = \frac{-\varepsilon\varphi\theta \cdot \sigma\upsilon\nu\theta}{\sigma\varphi\theta \cdot \eta\mu\theta} = -\frac{\eta\mu\theta}{\sigma\upsilon\nu\theta} \overset{*}{=} -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$* \eta\mu^2\theta + \sigma\upsilon\nu^2\theta = 1 \Leftrightarrow \sigma\upsilon\nu^2\theta = 1 - \frac{1}{4} \Leftrightarrow \sigma\upsilon\nu\theta = -\frac{\sqrt{3}}{2}$

Δ2 $$\frac{(1+\sigma\upsilon\nu\varphi)^2 + \eta\mu^2\varphi}{\eta\mu\varphi(1+\sigma\upsilon\nu\varphi)} = \frac{1 + 2\sigma\upsilon\nu\varphi + 1}{\eta\mu\varphi(1+\sigma\upsilon\nu\varphi)} = \frac{2(1+\sigma\upsilon\nu\varphi)}{\eta\mu\varphi(1+\sigma\upsilon\nu\varphi)} = \frac{2}{\eta\mu\varphi}$$

Δ3 · $\varepsilon\varphi\left(-\frac{31\pi}{4}\right) = -\varepsilon\varphi\left(\frac{32\pi}{4} - \frac{\pi}{4}\right) = -\varepsilon\varphi\left(-\frac{\pi}{4}\right) = \varepsilon\varphi\frac{\pi}{4} = 1$

· $\eta\mu\frac{29\pi}{6} = \eta\mu\left(\frac{30\pi}{6} - \frac{\pi}{6}\right) = \eta\mu\left(5\pi - \frac{\pi}{6}\right) = \eta\mu\left(\pi - \frac{\pi}{6}\right) = \eta\mu\frac{\pi}{6} = \frac{1}{2}$

α) $f(x) = x^2 + \sigma\upsilon\nu x + \frac{1}{2}$. β) $f(-x) = f(x)$ ΑΡΤΙΑ

γ) Θεωρούμε $g(x) = f(x) - 2024$ η οποία ΕΙΝΑΙ ΑΡΤΙΑ

ΕΙΝΑΙ $g(-x_1) = g(x_1) = 0 \Leftrightarrow g(x_1) = 0$ άρα $x_1 + x_2 = x_1 - x_1 = 0$